Maróthi György Memorial Competition, November 6 – December 8, 2023.

– Rules –

Description

The Institute of Mathematics of the University of Debrecen announces a competition in Mathematics for BSc students of the University of Debrecen in their first or second year of studies during the fall semester of the academic year 2023/2024. The competition is individual, registration is not required. The list of problems is published at noon on November 6, on the web page of the Institute:

`https://math.unideb.hu`

Available at: Hallgatóknak » Tehetséggondozás » Versenyek » Maróthi György Memorial Competition

Organizers

dr. Boros Zoltán	*(Institute Coordinator of Talent Management, Department of Analysis)*
dr. Bessenyei Mihály	*(Competition Secretary, Department of Analysis)*
dr. Remete László	*(Department of Algebra and Number Theory)*
dr. Szilasi Zoltán	*(Department of Geometry)*

Sponsorship

Organizers thank the financial support by the Morgan Stanley Magyarország Elemző Kft.

Formal requirements

Solutions to distinct problems should be elaborated on separate sheets of paper. Write your name, major, year, neptun code and the number of the problem which is elaborated on that sheet to the top of the page. The pdf file of the hand written solutions have to be sent by email to Zoltán Boros and Mihály Bessenyei:

`zboros@science.unideb.hu` and `besse@science.unideb.hu`.

Deadline for submission: December 8 (Friday), 2023, 12:00.

Ethical regulation

Though all problems can be solved using standard college mathematics, you can use any additional sources if it is appropriately cited in your solution. Cooperation of the participants (with each other or with any other person on any platform) is not allowed. If such a cooperation is established, all involved participants will be disqualified.

Every participant will be notified of his/her result.

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– Problems –

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1. Problem. *Find the minimum of the expression*

$$M = \frac{x_1}{S - x_1} + \dots + \frac{x_n}{S - x_n},$$

where $S = x_1 + \dots + x_n$ and $x_1, \dots, x_n$ are postitve reals!

(Posed by Bessenyei Mihály)

2. Problem. *For given $\alpha, \beta \in \mathbb{R}$ and $H \subseteq \mathbb{R}$, define $\alpha H + \beta := \{\alpha x + \beta \mid x \in H\}$. Find the smallest set $H \subseteq \mathbb{R}$ that contains zero and fulfills the equation*

$$H = \frac{1}{10} H \cup \left(\frac{1}{10} H + \frac{9}{10} \right).$$

(Posed by Bessenyei Mihály)

3. Problem. *Show that there is no infinite non-constant sequence of integers such that every element (starting from the second) is the harmonic mean of its two neighbours.*

(Posed by Remete László)

4. Problem. *Find all triples of consecutive integers, such that if you form their quotient in all possible ways, the sum of the 6 fractions is an integer.*

(Posed by Remete László)

5. Problem. *Let the endpoints of a body diagonal of a unit cube be A and B. Consider the sphere that is tangent to all three faces of the cube that contains A and all three edges of the cube that contains B. What is the radius of the sphere and what is the distance of its center from A and B?*

(Posed by Szilasi Zoltán)

6. Problem. *Let ABC be a triangle. Prove that if there exists a point D on the side AB such that CD is the geometric mean of AD and BD, then $\sin\alpha \sin\beta \le \sin^2 \frac{\gamma}{2}$ (where α, β, γ are the angles at A, B and C).*

(Posed by Szilasi Zoltán)

Solution to each problem is evaluated up to 5 points.
The order of the problems need not indicate their difficulty.